Math 251 **Section 4.1 Overview**

It is possible to learn many of the important attributes of a given (continuous) function, by examining its first and second derivatives. Some of these attributes are listed below:

1) **Intervals on which the function is either increasing or decreasing**
   a) If $f'>0$ on an interval, then *f* is increasing on that interval
   b) If $f'<0$ on an interval, then *f* is decreasing on that interval

2) **Intervals on which the function is either concave up or concave down.**
   a) If $f''>0$ on an interval, then *f* is concave up on that interval
   b) If $f''<0$ on an interval, then *f* is concave down on that interval

3) **Finding local maxima and minima (a.k.a. local extrema)**
   a) Find all critical points (points where $f' = 0$ or f' is undefined)

   b) Classify each critical point as a local max, local min, or neither

      i) If $f'=0$, then try using the <u>*2nd derivative test*</u>.
         (1) If $f' = 0$ and $f''>0$, then the critical point is a *local minimum*
         (2) If $f'=0$ and $f''<0$, then the critical point is a *local maximum*
         (3) If $f''=0$, then the test is inconclusive (so use the *1st derivatives test* below)

      ii) <u>*The 1st Derivatives Test*</u>: If f' is *undefined* or $f''=0$, then it is necessary to classify the point by examining if f' changes signs at the critical point.

         The critical points break the real number line into intervals on which f' is either positive or negative. So, to determine the sign of f' on a given interval, it is only necessary to test one point in that interval.

            (a) If f' changes from – to + at *x*, then there is a local minimum at *x*.
            (b) If f' changes from + to – at *x*, then there is a local maximum at *x*.
            (c) If f' does not change sign at *x*, then there is no min or max at *x*.

4) **Finding Global Extrema on a <u>closed domain</u> [a,b]**
   In section 4.1 we only look at finding global extrema on one type of domain. We will deal with other domains in section 4.2

   On a <u>closed</u> domain [a,b], Global Extrema of a differentiable function can only occur at critical points or endpoints.
      a) Find all critical points of *f*.
      b) Evaluate *f* at all critical points and both endpoints to determine the global extrema.

5) **Inflection points – points where *f* changes concavity (i.e. where f'' changes sign)**

   To find inflection points, first solve $f''(x)=0$. These solutions are *possible* inflection points. These solutions also break the real-line into intervals on which f'' is either positive or negative.

   a) If f'' changes sign at one of these points, then that point is an inflection point.
   b) If f'' does not change sign at one of these points, then it is not an inflection point.